Jacques MARTINET June 6th, 2014

Comment on

Sur les corps résolubles de degré premier
(*On solvable fields of prime degree*)

by Soun-HiKWON and Jacques MARTINET,

Crelle **375/376** (1987), 12–23.

This paper was intended to find using Class Field Theory the minimal discriminants of some solvable fields of prime degree, and as far as possible in those days, to construct corresponding polynomials using Kummer Theory. Calculations were done essentially by hand, with the help of a *HP* pocket computer for numerical outputs.

Reducing polynomials by hand left us for the "smallest" field K of degree 5, signature $(1,2)$ and Galois type Aff_5 (group of order 20), for which $d_K = 35152 = 2^4 \cdot 13^3$, with the ugly polynomial

$$P_0 = X^5 - 2X^4 - 4X^3 - 96X^2 - 352X - 568\,,$$

of discriminant

$$d_P = 978758033723392 = d_K \cdot (166864)^2\,.$$

A few months after the paper appeared, Francisco DIAZ y DIAZ pointed out to us that we could have obtained a more convenient polynomial by reducing the quadratic form $\mathrm{Tr}(x\overline{x})$. This procedure is nowadays implemented in *PARI-GP* (the *polred* algorithm).

One of the polynomials that *polred* outputs is

$$P = X^5 - X^4 + 4X^3 - 2X^2 + X - 1\,,$$

the discriminant of which is equal to $d_K = 35152$.

Much progress has been done on the question of small discriminants since our paper was published, relying on computational techniques dealing with polynomials, no longer using CFT nor Kummer T.