APPENDIX TO SMALL DISCRIMINANTS, CLASS NUMBERS, AND THE GEOMETRY OF NUMBERS

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On Thursday, July 12th, 2012, during the congress held in Bordeaux for Sir Martin Taylor's sixtieth birthday, I met Sir Peter Swinnerton-Dyer, with whom I discussed on the problems I considered in my text *Small Discriminants, Class Numbers, and the Geometry of Numbers.* Essentially I asked him various questions of a historical nature.

1. My first question was relative to the lattice constants of the Minkowski domains attached to fields of signature (r_1, r_2) in degree $n = r_1 + 2r_2 \geq 4$.

He answered that to his knowlege there had been attempts to find the lattice constant in he case $n = r_1 = 4$. "This was considered with classical methods by Davenport; unsuccessfully; and I tried myself to attack the question by a computer program; unsuccessfully."

He added that maybe the totally imaginary domain $(r_1, r_2) = (0, 2)$, which is more symmetric, could be considered.

2. My second question concerned diophantine approximations.

It is very well-known (see [Cas1]) that the theory of approximation of real numbers by rational numbers is closely related to the 2-dimensional Minkowski domain $|x_1 x_2| < 1$, the domain of real quadratic fields.

Similarly it is known that the theory of approximation of complex numbers by numbers of a given complex quadratic field is closely related to the 4-dimensional Minkowski domain $|z_1 z_2| < 4$ (the "4" by convenience) of complex quartic fields; here however one considers admissible lattices containing the image of the ring of integers of the given imaginary quadratic field; see the theses of Descombe and Poitou in the fifties. (The analogous condition is automatic in the previous case, modulo an automorphism of the domain.)

My question was whether it has been previously considered simultaneous approximations of r_1 real numbers and r_2 complex numbers by numbers of a given field of signature (r_1, r_2), a question connected with the Minkowski domains relative to the double signature $(2r_1, 2r_2)$; and

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in particular, whether there has been work on the approximation of two of real numbers by conjugates elements of a given real quadratic field .

The answer was “not to my knowledge”.

3. Finally, it seems that for the totally real cubic domain, the sequence of successive minima could tend to infinity (which would imply a bound in $o(disc^{1/2})$ for the norms of representatives of ideal classes). I asked whether there could be a Markoff-like phenomenon in the totally real quartic case. The answer was “It exists, but I do not remember the details”. (Thus there is no bound in $o(disc^{1/2})$, only the bound in $O(disc^{1/2})$, due to Minkowski.)