Jacques Martinet

Comparison between the French and English editions
of the books
Les réseaux parfaits des espaces euclidiens
and
Perfect Lattices in Euclidean Spaces

The Springer book ("Gundlehren" no. 327) *Perfect Lattices in Euclidean Spaces* stems from a previous French book, *Les réseaux parfaits des espaces euclidiens*, edited in 1996 by Masson (now Dunod) in Paris.

We describe below the main differences which exist between the French and the English editions of in this book.

Of course, references have been updated, and many errors, from mere slips to important ones, have been corrected. For these two kinds of changes, the reader is referred to the files *Bibliographie mise à jour du livre "Les réseaux parfaits des espaces euclidiens"* and *Erratum au livre "Les réseaux parfaits des espaces euclidiens"*.

The more important modification concerns the Appendices. The four appendices of the French edition have been transformed into a single one (*Appendix 1 = Chapter 15: Semi-Simple Algebras and Quaternions*); this shortening has induced an enlargement of Section 1 of Chapter 8, and a new appendix (*Appendix 2 = Chapter 16: Strongly Perfect Lattices* has been written to account for Venkov's work connecting the theory of lattices with that of spherical designs. Another important change is the adjunction of a 7 th section to Chapter 9, in order to take into account recent progress obtained by Batut and Bavard on minimal classes and the classification of eutactic lattices.

As in the French edition, Propositions, Theorems, etc. are numbered without the number of the Chapter. The expression "Theorem ... is new" means that it does not exist in the French edition, not that it is claimed for originality.

Chapter 1

Definition 3.10, on *characteristic* or *parity vectors*, is new.
Proposition 1.5 is new; the previous Proposition 1.5 is now Proposition 1.6.
Section 8 on arbitrary sphere packings has been shortened.

Chapter 3

The notion of *strong eutaxy* has been introduced within Definition 2.2, and made more precise in the two displayed formulae 2.2' and 2.2".

Corollary 6.7 has been generalized and transformed into the more general Theorem 6.7.

Exercises 1.6 (on the trace of a product of projections), 2.1, and 2.2 are new; a new Question 1 has been inserted in Exercise 8.4.

Chapter 4

Section 8 has been modified: some parts have been shortened, and some complementary details have been added at the end of the section.

Exercises 2.5, 2.6, 3.5, 4.7, 4.8, and 4.9 are new; a new Question 2 has been added to Exercise 3.1 and Questions 5 and 6 in Exercise 4.2 are new.

Chapter 5

Section 5 has been shortened.

Definition 7.5 has been modified by the introduction of no. 7.5'.

Exercises 5.2, 7.1, and 7.2 are new; Exercise 7.1 is now Exercise 7.3; the exercise in Section 6 (previously Exercise 6.1) has been suppressed.

Chapter 6

Section 4 has been completely rewritten form Corollary 4.4 on, as well as the corresponding exercises 4.1 and 4.2.

Exercises 1.2 after Batut and Venkov, and 5.2 after Watson have been added.

Chapter 7

The last eleven lines of Section 2 have been suppressed; as a consequence, a shifting in the numbering occurs from no 2.4 on.

The last paragraph of Section 8 (translation in terms of lattices) has been suppressed.

Exercise 2.1 is new.

Chapter 8

Section 1 has been rewritten to take into account the concatenation of the four appendices into a single one. The notions of modularity over imaginary quadratic and totally definite quaternion orders with center $\mathbb{Z}$ have been made precise. To this end, Exercise 1.2 has been extended.

In Sections 2 and 3, Definitions 2.3' and 3.1' have been added to make more precise Definitions 2.3 and 3.1.

The part of Section 3 which concerns quaternionic laminations has been rewritten; Exercise 3.2 is now included in an enlarged Exercise 3.1.

In Section 4, the proof of Theorem 4.2 has been corrected; Remark 4.9 is new.

Section 7 now includes the identification of K'_{16} as the orthogonal in Leech of K'_8.

Section 8 has been made more formal by making use of the notion of a lattice over the ring of Gaussian integers.

Exercises 3.1 and 3.2 have been modified; see above; Exercise 4.2 has been enlarged, Exercises 4.3, 4.4 and 4.5 (on the lattices L_n) and 5,6, 5.7 and 5.8 (on K_{12} and some generalizations) are new.

Chapter 9

Section 1 has been extended by the adjunction of Definition 1.13 and Propositions 1.14 and 1.15 and the extension of Definition 1.1.

Theorem 6.10, which asserts that the set of dual-eutactic lattices is finite, is new (and original, though now included in a range of finiteness results proved in Bavard's preprint [Bav4]).

Section 7, devoted to recent results of Batut and Bavard, is new.

Exercise 5.1 and Question 6 in Exercise 6.1 are new; Exercise 6.2 has been corrected (the *ccc* alias *mcc* lattice had been forgotten in the French edition).

Chapter 10

Formulae 3.7, 3.8, 3.9 are now Formulae 3.a, 3.b, 3.c; consequently, Corollaries 3.10 and 3.11 are now Corollaries 3.7 and 3.8.

Chapter 11

Table 4.6 is now Table 4.5'; consequently, 4.7 and 4.8 are shifted to 4.6 and 4.7.

The notion of the *level of a lattice* has been added in Definition 5.3; Remarks 5.6 and 5.7 at the end of Section 5 and Exercise 5.5 are new; the notes have been modified in relation with these transformations.

Chapter 13

The end of Section 5 takes into account the classification results obtained by Sigrist for 16 dimensional lattices.

Chapter 14

In Section 1, the 33 Gram matrices for perfect, 7-dimensional lattices have been suppressed.
Theorem 6.1 on $(s+s^*)_{\max}$ en dimension $n \leq 8$ is new (and original).

Appendices

Appendix 1 is a shortened version of the four appendices of the French edition. Appendix 2 is new.