Jacques Martinet Put on line on February 6th, 2016

NUMERICAL DATA (Données numériques)

At the request of some readers (cf. [Bc4], p. 654, l. 3) I have extracted the following TEX-data from Chapter XIV of the 1996 French edition [Mar] of my book *Perfect Lattices in Euclidean Spaces*. I display below

(1) Gram matrices for perfect lattices in dimensions $n \le 7$;

(2) some data on sublattices of the root lattice $\mathbb{E}_8$.

The data (1) up to dimension **8** can be downloaded in *PARI*-format from my home page, in the *Catalogue of Perfect Lattices*.

1. Perfect lattices up to dimension 7. I have displayed below Gram matrices for the 48 perfect lattices of dimension $n \le 7$. These matrices are LLL-reduced. (For the definition of LLL-reduction, see the original paper [LLL] of Lenstra-Lenstra-Lovász, or Henri Cohen's book [Coh], ch. 2, § 2.6).

To choose the somewhat reduced matrices below has here the interest of getting in all cases matrices in bases of minimal vectors, which is often useful for computations. However we do not see on matrices roots nor automorphisms, for which we refer to Conway-Sloane's [C-S5].

The matrices below have been obtained either from matrices constructed previously in my book or from matrices of [C-S5].

$$P_1^1 = \begin{pmatrix} 1 \end{pmatrix} \qquad P_2^1 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \qquad P_3^1 = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

$$P_4^1 = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \end{pmatrix} \quad P_4^2 = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix}$$

$$P_5^1 = \begin{pmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{pmatrix} \quad P_5^2 = \begin{pmatrix} 4 & -2 & -2 & 1 & 1 \\ -2 & 4 & 1 & -2 & 1 \\ -2 & 1 & 4 & 1 & -2 \\ 1 & -2 & 1 & 4 & -2 \\ 1 & 1 & -2 & -2 & 4 \end{pmatrix} \quad P_5^3 = \begin{pmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{pmatrix}$$

$$P_6^1 = \begin{pmatrix} 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 & 1 & 2 \end{pmatrix} \quad P_6^2 = \begin{pmatrix} 4 & -2 & 1 & -2 & -2 & -1 \\ -2 & 4 & -2 & 1 & 1 & -1 \\ 1 & -2 & 4 & -2 & -2 & 2 \\ -2 & 1 & -2 & 4 & 1 & -1 \\ -2 & 1 & -2 & 1 & 4 & -1 \\ -1 & -1 & 2 & -1 & -1 & 4 \end{pmatrix}$$

$$P_6^3 = \begin{pmatrix} 2&1&1&1&1&1\\ 1&2&1&0&1&1\\ 1&1&2&1&1&1\\ 1&0&1&2&1&1\\ 1&1&1&1&2&1\\ 1&1&1&1&1&2 \end{pmatrix} \quad P_6^4 = \begin{pmatrix} 4&2&1&2&2&2\\ 2&4&2&1&1&2\\ 1&2&4&2&2&2\\ 2&1&2&4&1&2\\ 2&1&2&1&4&2\\ 2&2&2&2&2&4 \end{pmatrix} \quad P_6^5 = \begin{pmatrix} 4&2&2&1&2&2\\ 2&4&2&2&1&2\\ 2&2&4&0&2&1\\ 1&2&0&4&1&2\\ 2&1&2&1&4&0\\ 2&2&1&2&0&4 \end{pmatrix}$$

$$P_6^6 = \begin{pmatrix} 4&2&1&2&2&2\\ 2&4&2&1&2&2\\ 1&2&4&2&2&2\\ 2&1&2&4&2&2\\ 2&2&2&2&4&1\\ 2&2&2&2&1&4 \end{pmatrix} \quad P_6^7 = \begin{pmatrix} 2&1&1&1&1&1\\ 1&2&1&1&1&1\\ 1&1&2&1&1&1\\ 1&1&1&2&1&1\\ 1&1&1&1&2&1\\ 1&1&1&1&1&2 \end{pmatrix}$$

$$P_7^1 = \begin{pmatrix} 2&1&1&1&1&1&1\\ 1&2&1&1&1&1&1\\ 1&1&2&0&1&1&1\\ 1&1&0&2&0&1&1\\ 1&1&1&0&2&1&1\\ 1&1&1&1&1&2&1\\ 1&1&1&1&1&1&2 \end{pmatrix} P_7^2 = \begin{pmatrix} 3&1&1&1&1&1&1\\ 1&3&-1&1&-1&1&1\\ 1&-1&3&1&1&-1&-1\\ 1&1&1&3&1&-1&-1\\ 1&-1&1&1&3&-1&-1\\ 1&1&-1&-1&-1&3&1\\ 1&1&-1&-1&-1&1&3 \end{pmatrix} P_7^3 = \begin{pmatrix} 4&2&1&2&2&2&2\\ 2&4&2&1&1&2&2\\ 1&2&4&2&2&2&2\\ 2&1&2&4&1&2&2\\ 2&1&2&1&4&2&2\\ 2&2&2&2&2&4&1\\ 2&2&2&2&2&1&4 \end{pmatrix}$$

$$P_7^4 = \begin{pmatrix} 2&1&1&1&1&1&1\\ 1&2&1&0&1&1&1\\ 1&1&2&1&1&1&1\\ 1&0&1&2&1&1&1\\ 1&1&1&1&2&1&1\\ 1&1&1&1&1&2&1\\ 1&1&1&1&1&1&2 \end{pmatrix} \quad P_7^5 = \begin{pmatrix} 4&2&2&2&1&-1&1\\ 2&4&2&2&-1&1&-1\\ 2&2&4&1&-1&-1&1\\ 2&2&1&4&-1&1&-1\\ 1&-1&-1&-1&4&-2&0\\ -1&1&-1&1&-2&4&-2\\ 1&-1&1&-1&0&-2&4 \end{pmatrix}$$

$$P_7^6 = \begin{pmatrix} 6&3&3&2&3&-1&-1\\ 3&6&3&3&2&1&1\\ 3&3&6&3&1&2&2\\ 2&3&3&6&3&-1&-1\\ 3&2&1&3&6&-3&-3\\ -1&1&2&-1&-3&6&2\\ -1&1&2&-1&-3&2&6 \end{pmatrix} \quad P_7^7 = \begin{pmatrix} 4&2&2&1&2&2&2\\ 2&4&2&2&1&1&0\\ 2&2&4&2&0&2&1\\ 1&2&2&4&-1&0&-1\\ 2&1&0&-1&4&0&2\\ 2&1&2&0&0&4&2\\ 2&0&1&-1&2&2&4 \end{pmatrix}$$

$$P_7^8 = \begin{pmatrix} 4&2&2&-1&2&-2&-2\\ 2&4&0&1&1&-1&-2\\ 2&0&4&0&0&0&0\\ -1&1&0&4&-2&2&1\\ 2&1&0&-2&4&-1&-1\\ -2&-1&0&2&-1&4&1\\ -2&-2&0&1&-1&1&4 \end{pmatrix} \quad P_7^9 = \begin{pmatrix} 10&5&3&3&3&3&3\\ 5&10&-2&5&5&5&5\\ 3&-2&10&3&3&3&3\\ 3&5&3&10&3&3&3\\ 3&5&3&3&10&3&3\\ 3&5&3&3&3&10&5\\ 3&5&3&3&3&5&10 \end{pmatrix}$$

$$P_7^{10} = \begin{pmatrix} 4&2&-2&2&-1&2&2\\ 2&4&-2&1&1&2&2\\ -2&-2&4&0&-1&-1&0\\ 2&1&0&4&0&2&2\\ -1&1&-1&0&4&-1&0\\ 2&2&-1&2&-1&4&2\\ 2&2&0&2&0&2&4 \end{pmatrix} \quad P_7^{11} = \begin{pmatrix} 6&3&3&2&3&2&-3\\ 3&6&2&-1&2&3&-3\\ 3&2&6&3&0&3&-1\\ 2&-1&3&6&1&1&1\\ 3&2&0&1&6&-1&-3\\ 2&3&3&1&-1&6&-2\\ -3&-3&-1&1&-3&-2&6 \end{pmatrix}$$

$$P_7^{12} = \begin{pmatrix} 6&3&2&2&2&2&2\\ 3&6&-1&3&3&3&3\\ 2&-1&6&2&2&2&2\\ 2&3&2&6&2&2&2\\ 2&3&2&2&6&2&2\\ 2&3&2&2&2&6&2\\ 2&3&2&2&2&2&6 \end{pmatrix} \quad P_7^{13} = \begin{pmatrix} 8&4&3&-3&4&3&-3\\ 4&8&4&1&3&4&-4\\ 3&4&8&-3&4&1&-1\\ -3&1&-3&8&-4&-1&-2\\ 4&3&4&-4&8&4&1\\ 3&4&1&-1&4&8&-3\\ -3&-4&-1&-2&1&-3&8 \end{pmatrix}$$

$$P_7^{14} = \begin{pmatrix} 6&-3&-3&3&2&-2&2\\ -3&6&3&0&-3&3&1\\ -3&3&6&-2&-3&1&1\\ 3&0&-2&6&-1&-2&0\\ 2&-3&-3&-1&6&-2&0\\ -2&3&1&-2&-2&6&2\\ 2&1&1&0&0&2&6 \end{pmatrix} \quad P_7^{15} = \begin{pmatrix} 8&-4&-4&-3&1&-4&-4\\ -4&8&4&4&2&1&0\\ -4&4&8&4&3&3&1\\ -3&4&4&8&-1&4&2\\ 1&2&3&-1&8&0&-4\\ -4&1&3&4&0&8&4\\ -4&0&1&2&-4&4&8 \end{pmatrix}$$

$$P_7^{16} = \begin{pmatrix} 6&3&2&-2&3&-2&-3\\ 3&6&-1&-3&1&1&-3\\ 2&-1&6&-2&3&0&1\\ -2&-3&-2&6&-3&-2&1\\ 3&1&3&-3&6&1&-2\\ -2&1&0&-2&1&6&-1\\ -3&-3&1&1&-2&-1&6 \end{pmatrix} \quad P_7^{17} = \begin{pmatrix} 6&2&-2&3&-2&-2&2\\ 2&6&-3&1&2&2&3\\ -2&-3&6&1&-1&-1&-3\\ 3&1&1&6&-3&-3&2\\ -2&2&-1&-3&6&2&-1\\ -2&2&-1&-3&2&6&-1\\ 2&3&-3&2&-1&-1&6 \end{pmatrix}$$

$$P_7^{18}=\begin{pmatrix} 8 & -4 & 4 & 4 & 3 & 1 & -3 \\ -4 & 8 & 0 & -4 & 1 & -3 & 1 \\ 4 & 0 & 8 & 3 & 4 & -3 & -4 \\ 4 & -4 & 3 & 8 & -1 & -1 & -4 \\ 3 & 1 & 4 & -1 & 8 & 1 & -3 \\ 1 & -3 & -3 & -1 & 1 & 8 & -1 \\ -3 & 1 & -4 & -4 & -3 & -1 & 8 \end{pmatrix} \quad P_7^{19}=\begin{pmatrix} 6 & 3 & 3 & 3 & -2 & -3 & 3 \\ 3 & 6 & 3 & 0 & -2 & 0 & 0 \\ 3 & 3 & 6 & 0 & -3 & -2 & 2 \\ 3 & 0 & 0 & 6 & -2 & -1 & 1 \\ -2 & -2 & -3 & -2 & 6 & -1 & -2 \\ -3 & 0 & -2 & -1 & -1 & 6 & -3 \\ 3 & 0 & 2 & 1 & -2 & -3 & 6 \end{pmatrix}$$

$$P_7^{20}=\begin{pmatrix} 6 & 3 & -3 & -2 & 3 & -2 & -3 \\ 3 & 6 & -3 & -3 & 1 & 1 & -3 \\ -3 & -3 & 6 & 0 & 0 & -1 & 1 \\ -2 & -3 & 0 & 6 & -3 & -1 & 3 \\ 3 & 1 & 0 & -3 & 6 & -2 & -3 \\ -2 & 1 & -1 & -1 & -2 & 6 & -1 \\ -3 & -3 & 1 & 3 & -3 & -1 & 6 \end{pmatrix} \quad P_7^{21}=\begin{pmatrix} 6 & 3 & 3 & -2 & -2 & -2 & -2 \\ 3 & 6 & 3 & -3 & 1 & -3 & -3 \\ 3 & 3 & 6 & 0 & 1 & -1 & 0 \\ -2 & -3 & 0 & 6 & 2 & 0 & 3 \\ -2 & 1 & 1 & 2 & 6 & -2 & 2 \\ -2 & -3 & -1 & 0 & -2 & 6 & 2 \\ -2 & -3 & 0 & 3 & 2 & 2 & 6 \end{pmatrix}$$

$$P_7^{22}=\begin{pmatrix} 4 & 2 & 2 & 1 & 1 & 2 & -2 \\ 2 & 4 & 1 & 2 & 2 & 0 & -2 \\ 2 & 1 & 4 & 2 & -1 & 2 & -1 \\ 1 & 2 & 2 & 4 & 1 & 0 & 0 \\ 1 & 2 & -1 & 1 & 4 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 & 4 & -1 \\ -2 & -2 & -1 & 0 & 0 & -1 & 4 \end{pmatrix} \quad P_7^{23}=\begin{pmatrix} 6 & 3 & 3 & 3 & 2 & -1 & -2 \\ 3 & 6 & 3 & 3 & -1 & 1 & -1 \\ 3 & 3 & 6 & 1 & 1 & -2 & 1 \\ 3 & 3 & 1 & 6 & 2 & 2 & 0 \\ 2 & -1 & 1 & 2 & 6 & -2 & -1 \\ -1 & 1 & -2 & 2 & -2 & 6 & -1 \\ -2 & -1 & 1 & 0 & -1 & -1 & 6 \end{pmatrix}$$

$$P_7^{24}=\begin{pmatrix} 6 & 3 & -2 & 3 & -2 & 1 & 1 \\ 3 & 6 & 1 & 1 & -3 & -1 & -1 \\ -2 & 1 & 6 & 1 & 2 & -3 & 1 \\ 3 & 1 & 1 & 6 & 1 & 1 & 0 \\ -2 & -3 & 2 & 1 & 6 & 1 & 2 \\ 1 & -1 & -3 & 1 & 1 & 6 & -2 \\ 1 & -1 & 1 & 0 & 2 & -2 & 6 \end{pmatrix} \quad P_7^{25}=\begin{pmatrix} 6 & 2 & 2 & 1 & 3 & 3 & 3 \\ 2 & 6 & -2 & 3 & 1 & 3 & -1 \\ 2 & -2 & 6 & -3 & -1 & 0 & 2 \\ 1 & 3 & -3 & 6 & 3 & 0 & 0 \\ 3 & 1 & -1 & 3 & 6 & 1 & 1 \\ 3 & 3 & 0 & 0 & 1 & 6 & 2 \\ 3 & -1 & 2 & 0 & 1 & 2 & 6 \end{pmatrix}$$

$$P_7^{26}=\begin{pmatrix} 4 & 2 & 1 & 2 & 2 & 2 & 2 \\ 2 & 4 & 2 & 1 & 1 & 2 & 2 \\ 1 & 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 1 & 2 & 4 & 1 & 2 & 2 \\ 2 & 1 & 2 & 1 & 4 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 4 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 4 \end{pmatrix} \quad P_7^{27}=\begin{pmatrix} 4 & 2 & 1 & 2 & 2 & 2 & -1 \\ 2 & 4 & 2 & 1 & 2 & 2 & 1 \\ 1 & 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 1 & 2 & 4 & 2 & 2 & 1 \\ 2 & 2 & 2 & 2 & 4 & 1 & 1 \\ 2 & 2 & 2 & 2 & 1 & 4 & 1 \\ -1 & 1 & 2 & 1 & 1 & 1 & 4 \end{pmatrix}$$

$$P_7^{28}=\begin{pmatrix} 4 & 2 & 2 & 2 & 1 & 2 & 1 \\ 2 & 4 & 2 & 1 & 1 & 0 & 2 \\ 2 & 2 & 4 & 1 & 2 & 1 & 1 \\ 2 & 1 & 1 & 4 & 2 & 2 & 2 \\ 1 & 1 & 2 & 2 & 4 & 0 & 2 \\ 2 & 0 & 1 & 2 & 0 & 4 & 1 \\ 1 & 2 & 1 & 2 & 2 & 1 & 4 \end{pmatrix} \quad P_7^{29}=\begin{pmatrix} 4 & 2 & 1 & 2 & 1 & 2 & 2 \\ 2 & 4 & 2 & 1 & 2 & 2 & 2 \\ 1 & 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 1 & 2 & 4 & 2 & 2 & 2 \\ 1 & 2 & 2 & 2 & 4 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 4 & 1 \\ 2 & 2 & 2 & 2 & 2 & 1 & 4 \end{pmatrix} \quad P_7^{30}=\begin{pmatrix} 4 & 2 & 2 & 2 & 1 & 2 & 1 \\ 2 & 4 & 2 & 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & 4 & 2 & 2 & 2 \\ 1 & 1 & 2 & 2 & 4 & 2 & 2 \\ 2 & 2 & 1 & 2 & 2 & 4 & 1 \\ 1 & 2 & 2 & 2 & 2 & 1 & 4 \end{pmatrix}$$

$$P_7^{31}=\begin{pmatrix} 4 & 2 & 2 & 2 & -1 & 2 & 1 \\ 2 & 4 & 2 & 2 & 1 & 1 & 1 \\ 2 & 2 & 4 & 2 & 1 & 1 & 2 \\ 2 & 2 & 2 & 4 & 1 & 2 & 2 \\ -1 & 1 & 1 & 1 & 4 & 1 & 2 \\ 2 & 1 & 1 & 2 & 1 & 4 & 2 \\ 1 & 1 & 2 & 2 & 2 & 2 & 4 \end{pmatrix} \quad P_7^{32}=\begin{pmatrix} 4 & 2 & 1 & 2 & 2 & 2 & 2 \\ 2 & 4 & 2 & 1 & 2 & 2 & 2 \\ 1 & 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 1 & 2 & 4 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 1 & 2 \\ 2 & 2 & 2 & 2 & 1 & 4 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 4 \end{pmatrix} \quad P_7^{33}=\begin{pmatrix} 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 \end{pmatrix}$$

2. Root Lattices. Korkine-Zolotareff bases for root lattices (ch. 4) are those having Gram matrices with 2 on the diagonal and 1 or 0 off the diagonal and the smallest possible number of 0 (none for $\mathbb{A}_n$, one pair for $\mathbb{D}_n$ and two pairs for $\mathbb{E}_n$). Matrices for $\mathbb{A}_n$ are LLL-reduced, the others are up to a permutation of basis vectors. In the case of $\mathbb{D}_n$ one obtains an LLL-reduced matrix by putting the zeros at places $(2,4)$ and $(4,2)$; these are the choices made in Section 1. The two matrices below, the second of which is LLL-reduced, represent $\mathbb{E}_8$.

$$\mathbb{E}_8 \ : \ \begin{pmatrix} 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \end{pmatrix} \quad , \quad \begin{pmatrix} 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \end{pmatrix} .$$

Deleting successively the last rows and columns, we obtain the sequences of cross-sections

$$\mathbb{E}_8 \supset \mathbb{E}_7 \supset \mathbb{E}_6 \supset \mathbb{E}_5 \simeq \mathbb{D}_5 \supset \mathbb{E}_4 \simeq \mathbb{A}_4 \supset \mathbb{E}_3 \simeq \mathbb{A}_2 \perp \mathbb{A}_1 \supset \mathbb{A}_1 \perp \mathbb{A}_1 \supset \mathbb{A}_1 \supset \{0\}$$

and

$$\mathbb{E}_8 \supset \mathbb{E}_7 \supset \mathbb{E}_6 \supset \mathbb{D}_5 \supset \mathbb{D}_4 \supset \mathbb{A}_3 \supset \mathbb{A}_2 \supset \mathbb{A}_1 \supset \{0\}\,,$$

respectively. All sections of the second matrix are LLL-reduced. We have found this way LLL-reduced matrices (up to a permutation) for all irreducible root lattices of dimension $n \leq 8$.

References

References are those of the updated bibliography (below in my homepage).

First part.

[Coh] H. Cohen, *A Course in Computational Algebraic Number Theory*, Graduate Texts in Math. **138**, Springer-Verlag, Heidelberg (1993).
[C-S5] J.H. Conway, N.J.A. Sloane, *Low-Dimensional Lattices. III. Perfect Forms*, Proc. Royal Soc. London **A 418** (1988), 43–80.
[Mar] J. Martinet, *Les réseaux parfaits des espaces euclidiens*, Masson, then Dunod, Paris (1996). [*English edition*: *Perfect Lattices in Euclidean Spaces*, Grundlehren **327**, Springer-Verlag, Heidelberg, 2003, 527 pp.]

Second part.

[Bc4] R. Bacher, *Construction of some perfect integral lattices with minimum* 4, J. Th. Nombres de Bordeaux **27** (2015), 655–687 ; preprint at arXiv:1401.3601 (15. Jan. 2014).