Automorphisms of Lattices. Application to Curves

Let E be a Euclidean space, let G be a finite subgroup of its orthogonal group and let $\mathcal{L}_G$ be the set of G-invariant lattices Λ in E. We compute for some low-dimensional examples the automorphism groups of the various lattices in $\mathcal{L}_G$, determine the subset of those for which there exists an isometry $u : \Lambda \to \Lambda^*$ commuting with G, and consider in particular symplectic G-lattices (those for which $u^2 = -\mathrm{Id}$).

Symplectic lattices correspond to complex, principally polarized Abelian varieties. Using Torelli's theorem, we construct the isometry class of lattices associated with some curves of genus 2 or 3, and obtain for some groups G information on curves admitting G as an automorphism group.

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