Warning concerning the slides of the talk

Hermite vs Minkowski

delivered at the C.I.R.M. congress of November 30th to December 4th, 2009

In a visit to Bordeaux in October 2007, Achill Schürmann suggested that the bound for the quotient H/M defined in the slides could be $\frac{n}{4}$ in dimensions n in the range n=4-8, attained uniquely on centred cubic lattices. A fortnight later I claimed the I could prove this conjecture, using the results on Watson's index theory that can be read in my 2001 paper at L'Enseignmement Mathématique. The idea was that I could easily reduce myself to the easier case of well-rounded lattices.

When I began writing (only last month) the corresponding paper, I realized that I could not give an *a priori* proof of this reduction to well-rounded lattices, and I had to go through a lot of complicated details, especially when n=8 and the index in the lattice Λ of a sublattice Λ' generated by successive minima is equal to 4 or 5.

At the time I am writing this note (*October 28th, 2012*), I have (essentially) written all details for what concerns dimensions $n \leq 8$.

Thus when I delivered the 2009 talk in Luminy the results were firmly established only in the case of well-rounded lattices and 2-elementary quotients Λ/Λ' .

The same remarks apply to the (unpublished) talk I delivered on July 12th, 2012 in Bordeaux at the congress held to celebrate Sir Martin Taylor's sixtieth birthday. I announced that I could prove the upper bound $H/M \leq \frac{n}{4} = \frac{9}{4}$ in dimension 9, with equality only on the centred cubic lattices and on the lifts of the unique binary [9, 2, 6]-code. On October 28th, 2012, the result is firmly established only in the following cases:

- Λ is well-rounded;
- Λ/Λ' is elementary (using special properties of binary codes);
- $\bullet \ [\Lambda : \Lambda'] \leq 4;$
- $[\Lambda : \Lambda'] \ge 10$ (because Λ is then necessarily well-rounded). Indices 5 to 9 need specific work which could prove difficult.

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